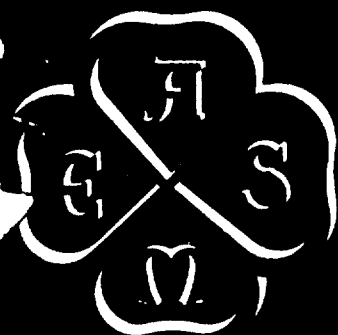


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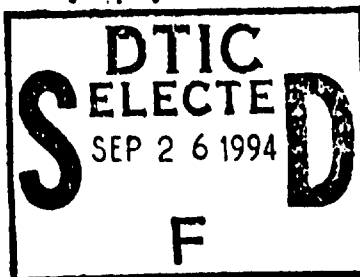
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## On the Generation of Noise by Turbulent Jets

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# ON THE GENERATION OF NOISE BY TURBULENT JETS

Alan Powell<sup>†</sup>

Elementary methods are used to provide a fundamental understanding of the salient features of jet noise. Simple reasoning yields the following results consistent with experiment:

1. Total noise power  $\sim (\text{jet velocity})^8 (\text{jet diameter})^2$ ,
2. Noise generation intensity nearly constant between the jet exit and the end of the potential cone, some distance after which it falls off very rapidly,
3. High-frequency spectrum level  $\sim (\text{jet velocity})^9 (\text{jet diameter})^1 \times (\text{frequency})^{-2}$ ,
4. Low-frequency spectrum level  $\sim (\text{jet velocity})^5 (\text{jet diameter})^5 \times (\text{frequency})^2$ .

It is shown how convection effects can be incorporated together with data from turbulence measurements. The failure of existing methods is illustrated by reference to rocket noise.

## 1. INTRODUCTION

To introduce the subject of the generation of noise by jets, it is desirable to outline the basis for what understanding we have of this very complex phenomena without getting too involved with theoretical intricacies. For this reason, elementary ideas are developed in such a way as to indicate the underlying rationale behind some of the salient features. In examining the results of these simple but sound notions we shall have cause to refer to experiment, particularly as a number of assumptions have to be made concerning the turbulent flow itself.

This paper is concerned with the noise generated by the turbulence of jets. Another aspect concerns the noise generated when a vortex, or turbulence, is swept through a shockwave as in the choked jets of air models, some jet engines, and rockets; or in supersonic wind tunnels and high-pressure valve gear. It does not seem to be generally the prime factor in the jets of propulsive units. Details of this aspect, and a wealth of other relevant information may be found in the References, especially (1)-(7).

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## 2. SOME GENERAL THEORETICAL CONSIDERATIONS

The approach will be to use physical arguments in an attempt to obtain a general form of results which display the principal observed characteristics of jet noise.

### The Simple Source

The most elementary type of source generator is the simple source, so this is a logical place to start. It arises when there is a "fluctuating source of matter", mathematically speaking. In practice, this could be a balloon inflated by an oscillatory air supply; in fact, the situation is not altered in principle if the balloon itself is removed. Both pressure and velocity fluctuations in the surrounding atmosphere are proportional to the rate of fluctuation of flow rate, i. e., to  $\rho u A f$ , where  $\rho$  is the fluid density,  $u$  is the mean velocity over an area  $A$  enclosing the source and  $f$  is the frequency. It turns out that the acoustic power, i. e., the power radiated away is

$$P_1 \sim (\rho u A)^2 f^2 / (\rho a), \quad (1)$$

where " $a$ " is the atmospheric speed of sound. Now  $(\rho u A)$  is the mass flow across the area  $A$ , or, effectively the rate of change of mass within  $A$ , so alternatively

$$P_1 \sim \left( \frac{dm}{dt} \right)^2 f^2 / (\rho a) \sim \left( \frac{d^2 m}{dt^2} \right)^2 / (\rho a), \quad (1a)$$

since differentiating a sinusoidally varying function amounts to multiplying by  $2\pi f$ .

Without committing ourselves about the details of the source, assume a simple similarity so that if  $U$  and  $D$  are typical of the velocities and dimensions of the system, then  $u \sim U$ ,  $A \sim D^2$  and  $f \propto U/D$ , giving from Equation (1)

$$P_1 \sim \rho U^4 D^2 / a \sim \rho U^3 D^2 (U/a). \quad (1b)$$

These results have been used satisfactorily in connection with the noise of pulse jets.<sup>8</sup>

### The Dipole Source

Next in order of complexity after the simple source is the dipole, constructed in principle by the placing of two simple sources, of opposite phase, at a very small distance,  $\delta$ , apart. To a first approximation the result is nothing, since one source simply ingests what fluid the other emits. So we must seek the next approximation. Now happenings at one source will be transmitted at the speed of sound to the other, i. e., at a time  $(\delta/a)$  later, during which time things have moved on there, in terms of phase, an amount  $(f\delta/a)$ . Hence the cancellation of pressure and of velocity are both imperfect by just this fraction, and so the acoustic power has an additional factor  $(f\delta/a)^2$ , becoming now

$$P_2 \sim (\rho u A \delta)^2 f^4 / (\rho a^3). \quad (2)$$

Note that  $(\rho u A \delta f)$  has the dimensions of rate of change of momentum,  $d(\mu)/dt$ , which is of course equitable to a force  $F$ ,

$$P_2 \sim \left[ \frac{d(\mu)}{dt} \right]^2 f^2 / (\rho a^3) \sim \left[ \frac{d^2(\mu)}{dt^2} \right]^2 / (\rho a^3) \sim \left( \frac{dF}{dt} \right)^2 / (\rho a^3). \quad (2a)$$

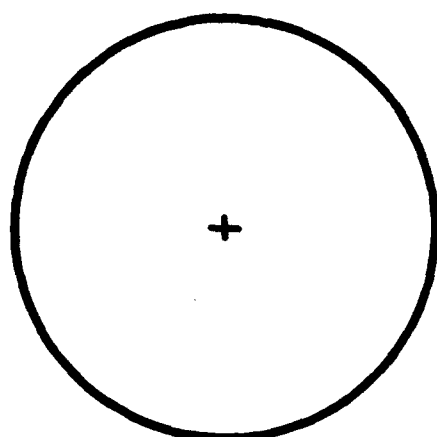
Again assuming simple geometric similarity (with  $\delta \sim D$ ),

$$P_2 \sim \rho U^6 D^2 / a^3 \sim \rho U^3 D^2 (U/a)^3. \quad (2b)$$

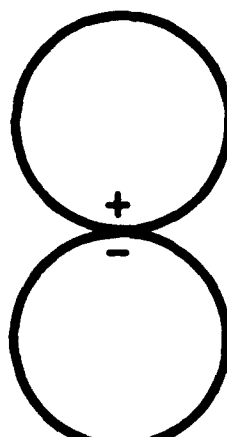
The origin of the momentum fluctuation (the source had mass fluctuation) is easily visualized in the case of a propeller, or of a vibrating wire, the air moving with rapidity from front to rear (like the doublet in aerodynamics, which is nothing more than a dipole of zero frequency). Of greater interest to the study of aerodynamic noise are those cases where fluctuating forces act between an unsteady fluid motion and a member which itself does not move. The best example is perhaps the reaction to the oscillating life on a circular cylinder which has an eddying wake. Measurements show that the radiation pattern is a figure of eight, which is what one would expect (see Figure 1). The acoustic power is found to vary very nearly with the sixth power of the stream velocity, at pertinent Reynold's numbers, in accordance with Equation (2b).

### The Quadrupole Source

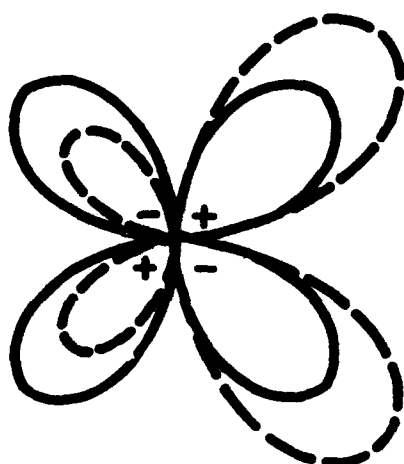
For ordinary jet noise, the fluctuations in the mean jet exit velocity are much too small to produce much simple source effect, or thrust fluctuations



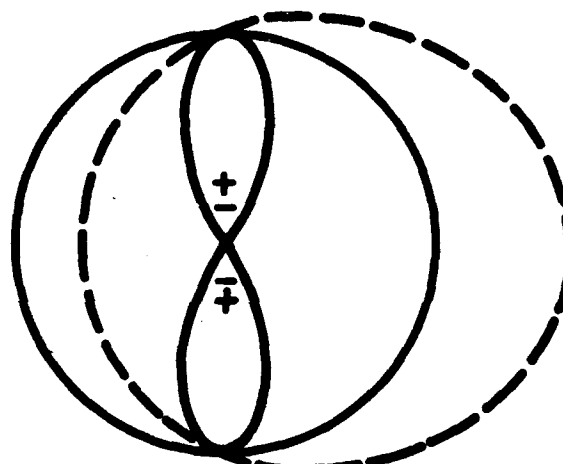
**SIMPLE SOURCE**



**DIPOLE**



**LATERAL**



**LONGITUDINAL**

**QUADRUPOLES**

**FIGURE 1. DIRECTIONALITY OF VARIOUS TYPES OF SOURCE. THE EFFECTS OF CONVECTION AT THE RELATIVELY SMALL VALUE OF  $M_c = 0.15$  ARE SHOWN ON THE LATERAL QUADRUPOLE AND ON A RANDOM ORIENTATION OF LONGITUDINAL QUADRUPOLES WHICH OTHERWISE WOULD BE UNIFORM IN ALL DIRECTIONS.**

for a dipole. We must, in fact, go just one step further, to the quadrupole source. Analogous to the formation of the dipole from a pair of out-of-phase simple sources, the quadrupole can be derived from a pair of oppositely phased dipoles. Thus again to a first approximation nothing is left (the net fluctuating force being zero) and the remnant has again the factor  $(f\delta/a)$  on both velocity and pressure, and, therefore,  $(f\delta/a)^2$  on sound power. Then

$$P_2 \sim (\rho u A \delta^2)^2 f^6 / (\rho a^5). \quad (3)$$

Here we note that  $(\rho u A \delta^2 f^2)$  has the dimensions of rate of change of momentum flux, call it  $d(\mu^2)/dt$ , so

$$P_3 \sim \left[ \frac{d(\mu^2)}{dt} \right]^2 f^2 / (\rho a^5). \quad (3a)$$

The rate of change of momentum flux will certainly not be zero in the turbulence of a jet. Therefore, we think of interpreting these results in terms of turbulent jets, or more precisely as we shall see, of an 'eddy'. Then proceeding just as before,

$$P_3 \sim \rho U^8 D^2 / a^5 \sim \rho U^3 D^2 (U/a)^5. \quad (3b)$$

The source, dipole and quadrupole are increasingly dependent on the Mach number, for since  $\rho U^3 D^2$  is proportional to the kinetic power of the system, the efficiencies of conversion to acoustic power are respectively like  $(U/a)$ ,  $(U/a)^3$  and  $(U/a)^5$ . The  $U^8 D^2$  of Equation (3b) is characteristic of jet noise which appears to emit from a mixture of lateral and longitudinal quadrupoles, the adjectives being suggestive of how the constituent dipoles are brought together, see Figure 1.

### 3. CONNECTION WITH LIGHTHILL'S THEORY

The idea that quadrupole sources should be the principal generators of noise in an unsteady flow, is the gist of Professor Lighthill's outstanding papers,<sup>9, 10</sup> in which he develops the mathematical details in a more rigorous manner. Here it will be shown how his results reduce to the forms just obtained. For example, an important form used by Proudman<sup>11</sup> for the noise

power  $P$  generated by a volume  $V$  of isotropically turbulent fluid of density  $\rho$  is

$$\frac{P}{V} = \frac{3}{4\pi\rho_a a_a^5} \int \left[ \frac{\partial^2(\rho u^2)}{\partial t^2} \right]_z \left[ \frac{\partial^2(\rho u^2)}{\partial t^2} \right]_{z=0} dV(z) \quad (4)$$

Here  $u$  is the turbulent velocity component in any direction,  $\rho_a$  and  $a_a$  are the density and speed of sound in the surrounding ambient atmosphere. Now each of the factors in the volume integral can be made nondimensional by dividing it by, say, its rms value. The resultant integral is then simply a volume-- which we can call the "volume  $v$  of a noise generating eddy". (Definition!). Hence

$$P \sim \left[ \frac{d^2(\rho u^2)}{dt^2} \right]^2 v V / (\rho_a a_a^5) . \quad (4')$$

Now put  $m = \rho v$  as the "mass of the eddy", loosely speaking. Then with  $N$  eddies in the volume  $V$ , ( $V = Nv$ )

$$P \sim N \left[ \frac{d^2(mu^2)}{dt^2} \right]^2 / (\rho_a a_a^5) , \quad (4a)$$

which is exactly like Equation (3a) except we now have  $N$  sources, and can distinguish between the two densities involved. Alternatively from Equation (4') we can get

$$P \sim \rho^2 u^4 f^4 v V / (\rho_a a_a^5) , \quad (4a')$$

which turns out to be a very useful form for our purposes.

Making use of the similarity idea, with  $v \sim D^3$ ,  $V \sim D^3$ ,

$$P \sim \rho_a (\rho_j / \rho_a^2)^\epsilon U_j^8 D^2 / a_a^5 . \quad (4b)$$

The efficiency,  $\eta \sim (\rho_j / \rho_a)^{\epsilon-1} (U_j / a_a)^5 . \quad (4c)$

Here the density ratio  $(\rho / \rho_a)^2$  has been replaced by  $(\rho_j / \rho_a)^\epsilon$  with a jet system

in mind, since  $\rho$  must always be between the extremes of the jet density at the exit,  $\rho_j$  and atmospheric,  $\rho_a$ . (Presumably the form  $\{\rho_a + k(\rho_j - \rho_a)\}^2 / \rho_a^2$  would be better, but this complication is not worthwhile, as heated jets do not possess a simple similarity.<sup>12</sup>)

All the foregoing equations resting on the ideal of simple similarity should be supplemented by a nondimensional function, say  $G$ , to account for all effects (e.g., of Mach or Reynold's number) not represented. Later we shall have to consider some of the constituents making up  $G$ .

#### 4. IMPLICATIONS CONCERNING SPECTRA

Some properties of the noise power spectrum will now be inferred, still working in a rather general fashion by applying the foregoing Equation (4a') to parts of a turbulent jet on the strength that it leads to good agreement with experiment via Equation (4b). But since the information being sought is more detailed, presumably it will be more susceptible to any inadequacies of the assumptions made. That function  $G$  is therefore always implied, though for brevity not written; and for conciseness, embraces within it the factor  $(\rho_j / \rho_a)^\epsilon$ .

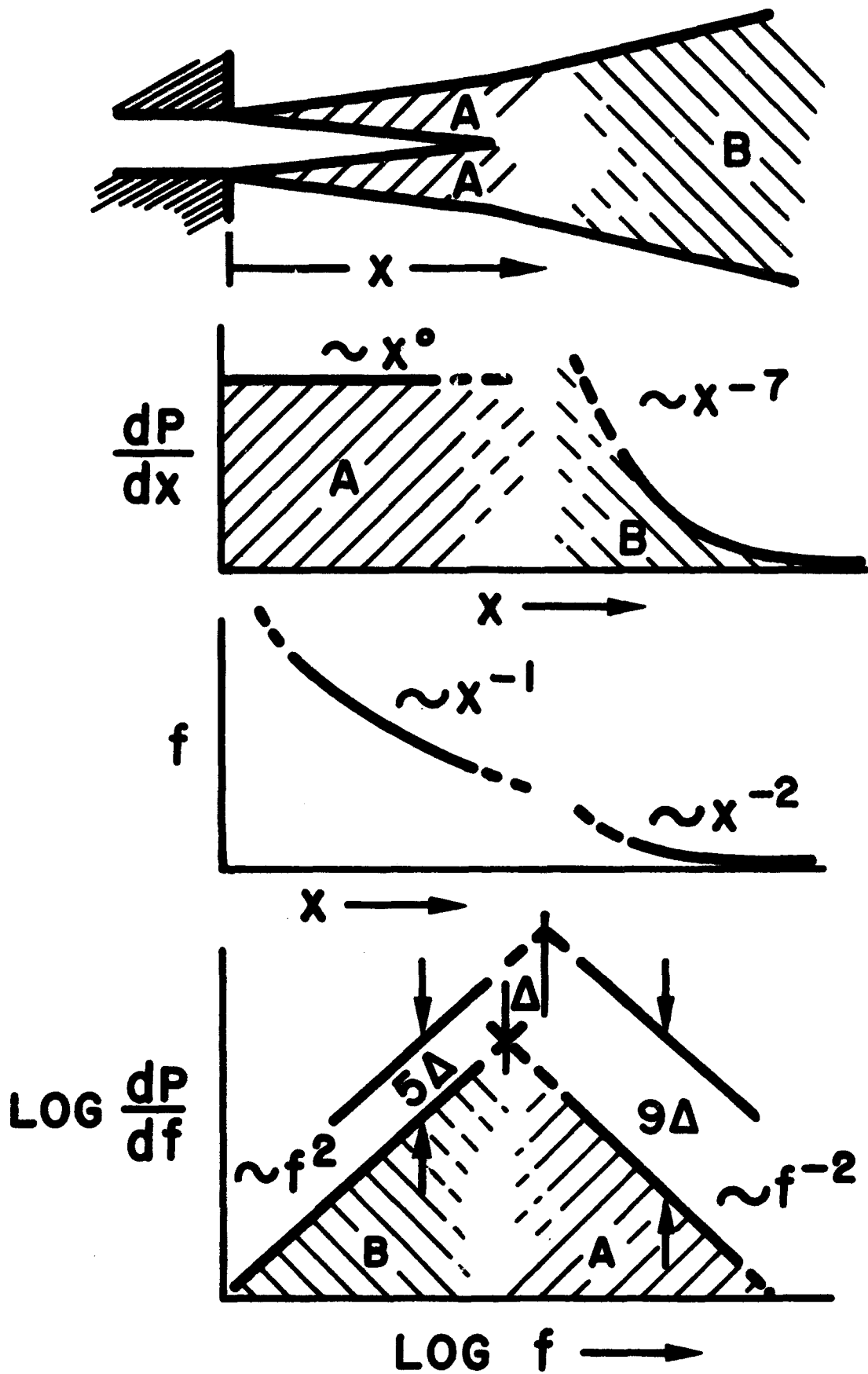
Consider the annular mixing region stretching from the orifice to the end of the potential cone where the turbulent front reaches the axis: call it Region A, see Figure 2. Simple similarity is then assumed to apply to all axial slices of thickness,  $dx$ , and area  $A$  of the turbulence, or more precisely, to those "noise producing eddies", which is a more lenient assumption. Then as  $V = A dx$ ,

$$(dP/dx)_A \sim \rho_a u^4 f^4 v A / a_a^5. \quad (5)$$

According to the assumed similarity, if  $\xi = x/D$ , then  $u \sim U_j$ ,  $f \sim U_j / D \xi$ ,  $v \sim D^3 \xi^3$ ,  $A \sim D^2 \xi$ . Hence Equation (5) yields for Region A

$$(dP/dx)_A \sim (\rho_a U_j^8 D / a_a^5) \xi^0. \quad (6a)$$

Thus the similarity argument leads to the very simple result that the generation intensity depends only on the total shear velocity  $U_j$  and not in any way upon the velocity gradients or upon the rate of spread of the mixing region. An exactly analogous situation holds for boundary layer noise.<sup>13</sup> At a short distance beyond the end of the potential cone the mean velocity distributions across the jet become similar in shape at all cross sections. Experimental results show that the



**FIGURE 2.** DIAGRAM ILLUSTRATING REGIONS A AND B, WITH TEN CORRESPONDING AXIAL DISTRIBUTIONS OF NOISE POWER GENERATION, FREQUENCIES VARIATION AND SPECTRAL CHARACTERISTICS (ILLUSTRATED FOR TWO VELOCITIES OF RATIO  $\Delta$ ).

maximum (mean) velocity, which naturally occurs on the axis, falls nearly inversely to the distance from the exit, while the width of the jet stream is proportional to it.<sup>14</sup> Thus here, in Region B,  $u \sim U_j/\xi$ ,  $f \sim U_j(D\xi^2)$ ,  $v \sim D^3\xi^3$  and  $A \sim D^2\xi^2$  so that

$$(dP/dx)_B \sim (\rho_a U_j^8 D/a_a^5) \xi^{-7} \quad (6b)$$

These two results imply that most of the noise power is generated at constant rate along the jet from the exit to the end of the potential cone, but soon after, it commences to fall off very rapidly, a point recently emphasized by Ribner who used similar arguments.<sup>15</sup> Further, since  $dP/df \sim (dP/dx)(dx/df)$  one can obtain information about the power spectrum. Thus

$$(dP/df)_A \sim (\rho_a U_j^9 D/a_a^5) f^{-2}, \quad \text{high freq} \quad (7a)$$

$$(dP/df)_B \sim (\rho_a U_j^5 D^5/a_a^5) f^2. \quad \text{low freq} \quad (7b)$$

Thus these simple similarity ideas yield a spectrum in which the low-frequency part increases with  $f^2$ , while the high-frequency part falls with  $f^{-2}$ , as shown in Figure 2.

It is important to note that neither part of the spectrum is to be expected to depend on  $U_j^8 D^2$ , even though the overall power does so.

The very highest frequencies cannot be expected to have been properly represented here, since the eddy size close to the exit unrealistically tends to zero. Also, closer to the end of the potential cone the similarity cannot hold. Further the transition process extends a considerable distance into Region B.<sup>14</sup>

Nevertheless, in all probability, the peak of the spectrum is associated with the region around the end of the potential cone, possibly a little downstream of it.

Of course, such a general approach cannot be expected to indicate how the properties of the two sections are to be joined together, nor their relative levels, nor whether or not they have overlapping frequencies. But we can define a peak at the intersection of the two slopes, for which

$$f_p \sim U_j/D \quad (8)$$

It is important to emphasize that it is far too restrictive to consider that just a single frequency emanates from each axial slice. We must assume that each slice generates a definite spectrum, which fulfills the similarity conditions in each region. It is clear that such spectra should fall away faster than  $f^2$  or  $f^{-2}$ , since otherwise a certain elaboration of the method is required.

By making no allowance for the roundness of the maximum to be expected, integration over the appropriate ranges gives

$$P = \frac{4}{3} (dP/df)_p f_p \simeq \frac{4}{3} (dP/dx)_A x_A \quad (9)$$

where  $(dP/df)_p$  is the peak value of the idealized triangular spectrum, and  $x_A$  is the length of Region A. The second part of Equation (9) is clearly very approximate.

Now retain the constants in Equation (5), and take the following very approximate but plausible values to calculate  $(dP/dx)_A$  at the end of the potential cone:

$u \simeq 0.15U_j$ ;  $v \simeq (D/5)^3$ ;  $f \simeq 0.2U_j/D$ ; and  $A = \pi D^2$ . Then Equation (9) gives  $P \simeq 5 \times 10^{-5} \rho_a U_j^8 D^2 / a_a^5$  in comforting likeness to Equation (11a), although we are here concerned only with orders of magnitude.

## 5. EXPERIMENTAL FINDINGS

### Overall Noise Power

The jet noise power is found to depend upon  $U_j^8 D^2$  in a rather remarkable manner, the index of  $U_j$  rarely being less than 7.5 or exceeding 8.5, even over a wide range of conditions, and all indications confirm the  $D^2$ . (See References 16-23.)

Figure 3 illustrates this for a series of N. A. C. A. measurements concerning model jets, unheated and heated to 1000°F, and full scale jet engines.<sup>19, 20</sup> To judge from this,

$$P \simeq k U_j^8 D^2, \quad k \simeq 5 \times 10^{-23} \text{ slugs sec}^5 \text{ ft}^{-8}, \quad (10)$$

holding over a range of nearly a million. These results vary very little for a range of  $10^{11}$ , the Reynold's number then extending from about  $10^5$  to  $50 \times 10^6$ , velocities from 175 to 2100 ft/sec, and temperatures from 400°R to 1600°R.

It is convenient to avoid a dimensional constant, so in view of the theoretical

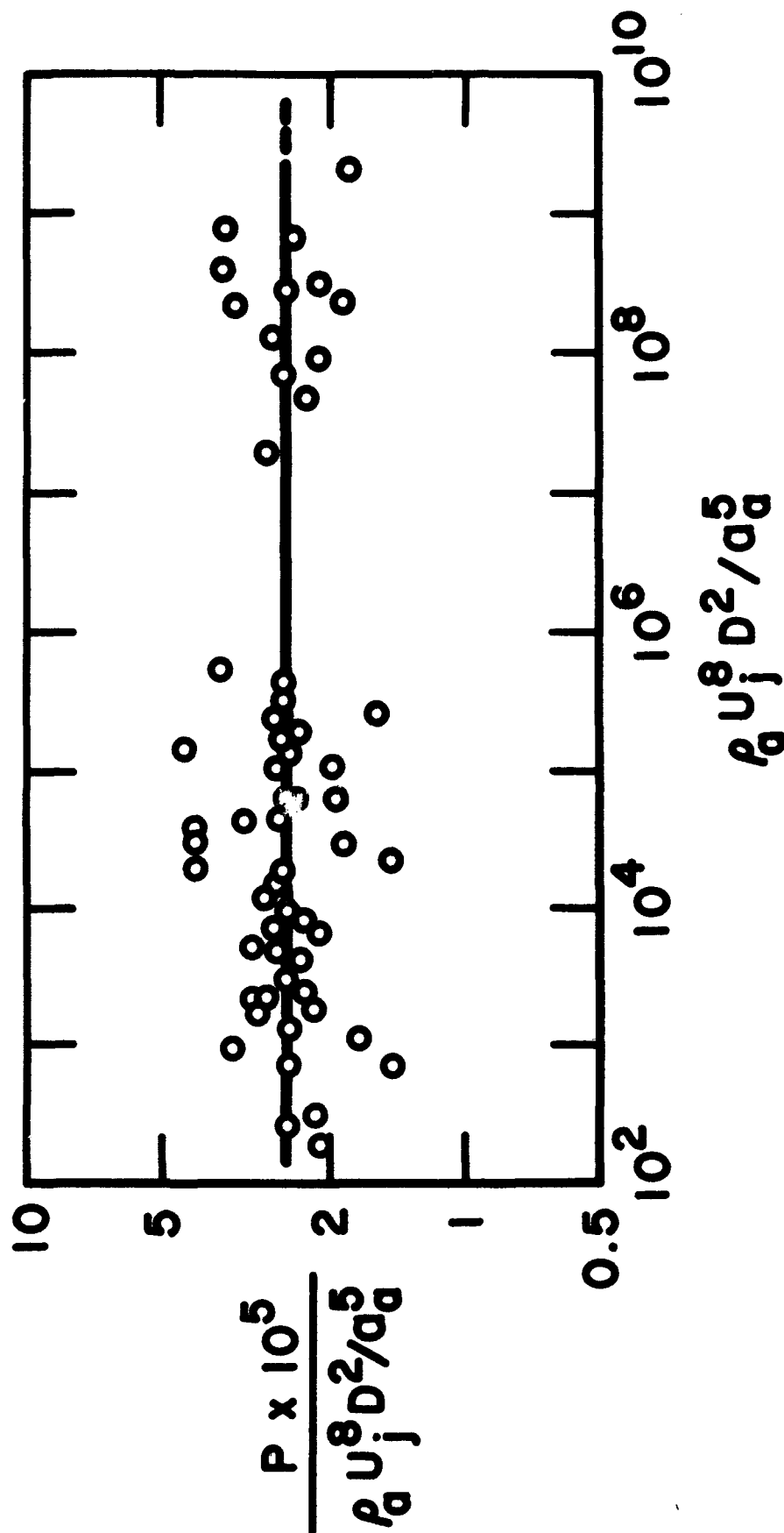


FIGURE 3. THIS DIAGRAM SHOWS HOW NEARLY IS THE ACOUSTIC POWER  $P$  PROPORTIONAL TO  $U_j^8 D^2$  OVER A RANGE OF NEARLY A MILLION. THE FACTOR  $\rho_a^5 / a_a^5$  IS A CONVENIENT CONSTANT. THE DATA IS TAKEN FROM N. A. C. A. WORK REPORTED BY CALLAGHAN AND COLES, AND BY ROLLIN AND CONCERNS UNHEATED AND HEATED MODEL JETS OF 5/16-, 3-, 4-, AND 5-INCHES IN DIAMETER AND THREE JET ENGINES.

reasoning, it can be made a factor of  $\rho_a/a_a^5$ . Then

$$P \simeq 3 \times 10^{-5} (\rho_a U_j^8 D^2 / a_a^5) G_1, \quad (11a)$$

with the efficiency increasing with jet temperature, since

$$\eta \simeq 7 \times 10^{-5} (\rho_a / \rho_j) (U_j / a_a)^5 G_1, \quad (11b)$$

where  $G_1$  is found to be a slowly varying function of all the independent variables such as velocity, temperature and their distributions, initial turbulence, swirl or minor pulsations, and noise. There is a little evidence suggesting that  $G_1 \sim (B_j/B_a)^2$  where  $B$  is the constant of  $p = \rho BT$ , on the basis of certain measurements using different gases.<sup>21</sup>

For the jet from established turbulent flow in a long pipe,  $G_1$  in Equation (11a) is about half,<sup>22</sup> while high initial turbulence increases it fourfold.<sup>21</sup>

Extensive efforts have been made to reduce the noise power by varying the nozzle configuration, without inducing an embarrassing thrust loss; reductions to less than a tenth seem difficult.

### Characteristics of Spectra

By far the most striking data in relation to the estimated spectrum shape are those by Cole and his colleagues.<sup>24</sup> They correlate spectra of model air jets, jet engines and rockets in the form of the single averaged curve shown in Figure 4. With it is a power spectrum curve for an engine,<sup>25</sup> and the spectrum estimated as above, drawn with its peak at the correct frequency location. They have been normalized by specifying equal areas under them. The correlation between them is remarkably good, though to some extent fortuitous. Below is a short account of relevant information that can be deduced from various published data.<sup>16, 17, 18, 20, 25, 26</sup>

Other data suggests that the high-frequency part of the spectrum may fall like the  $f^{-2}$  predicted, but frequently falls faster, especially at the highest frequencies and at high temperatures. The influence of velocity at fixed high frequencies certainly appears to be greater than as  $U_j^8$ , and possibly greater than the  $U_j^9$  of the simple theory. Although the data is not too precise, the spectrum level appears to depend on a little less than the first power of diameter.

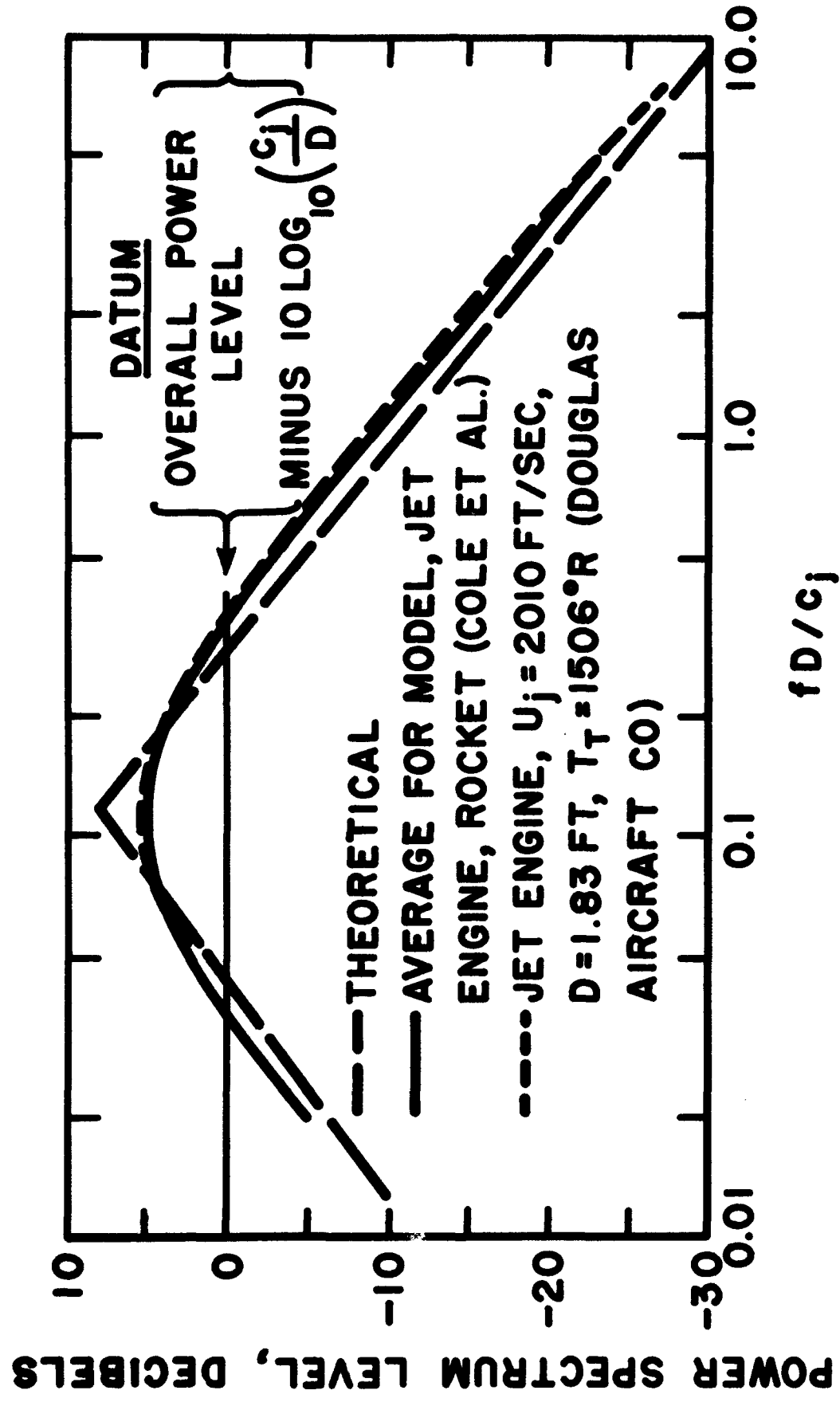


FIGURE 4. COMPARISON OF THEORETICAL SPECTRUM WITH AVERAGED SPECTRUM OF MODEL AIR JETS, JET ENGINES AND ROCKETS AND OF A TYPICAL JET ENGINE. THE LEVELS HAVE BEEN NORMALIZED BY REFERENCE TO THE OVERALL POWER LEVEL, AND THE PEAK OF THE THEORETICAL SPECTRUM IS PLACED AT THE CORRECT NONDIMENSIONAL FREQUENCY.

At low frequencies, the slope may be like  $f^2$ , or less steep according to several sources, but again becoming steeper with velocity and temperature. At fixed frequencies, the influence of velocity is a little greater than  $U_j^5$ , but certainly much less than  $U_j^8$ . The effects of diameter appear to be like  $D^4$  or  $D^{4.2/3}$ .

Because the spectral range is not always covered, and because not always are there sufficient straight line parts, it is not practical to define the peak frequency as in the theoretical considerations. Instead, we shall simply take the maximum of the spectrum. In Figure 4, the frequency scaling parameter is  $f_p D/c_j$  and, of course,  $c_j$  increases less rapidly than  $U_j$  for air jets, engines and rockets. This lack of proportionality with jet velocity is also in evidence elsewhere, and  $f_p \sim U_j^{1/2}$  seems a fair average, although evidence seems in accord that  $f_p \sim 1/D$ , approximately.

Thus, we see that our reasoning has lead to conclusions concerning the effect of the principal variables,  $U_j$ ,  $D$  and  $f$ , which certainly display the correct trends. On the whole, the results are very promising, considering the elementary nature of our derivation. Two shortcomings must be recognized however; first, we have not explained the gross directional characteristics of the radiated noise, and secondly, the failure of the peak frequency to scale directly with velocity casts some doubt on the accuracy of the picture of simple similarity particularly, in that the noise is generally thought to have suffered convection effects, the observed frequencies themselves being raised above those occurring in the flow. These deficiencies are the subjects of the next two sections, although the treatment is necessarily somewhat tentative until more data are available.

## 6. EFFECTS OF CONVECTION

One of the overt features of jet noise is that nearly all of it is radiated at a smallish acute angle to the direction of the jet as shown in Figure 5, which shows the directional pattern for a jet engine.<sup>25</sup> But the radiation associated with the form given in Equations (3b) or (4b) is symmetrical in upstream and downstream directions (whether the quadrupoles are lateral or longitudinal).

Lighthill explains this in a convincing manner. The noise-generating eddy system is obviously convected downstream with the local mean velocity, and what is more, it "knows" it. Then the cancellation effects we considered earlier in developing our general ideas are biased, for signals from source

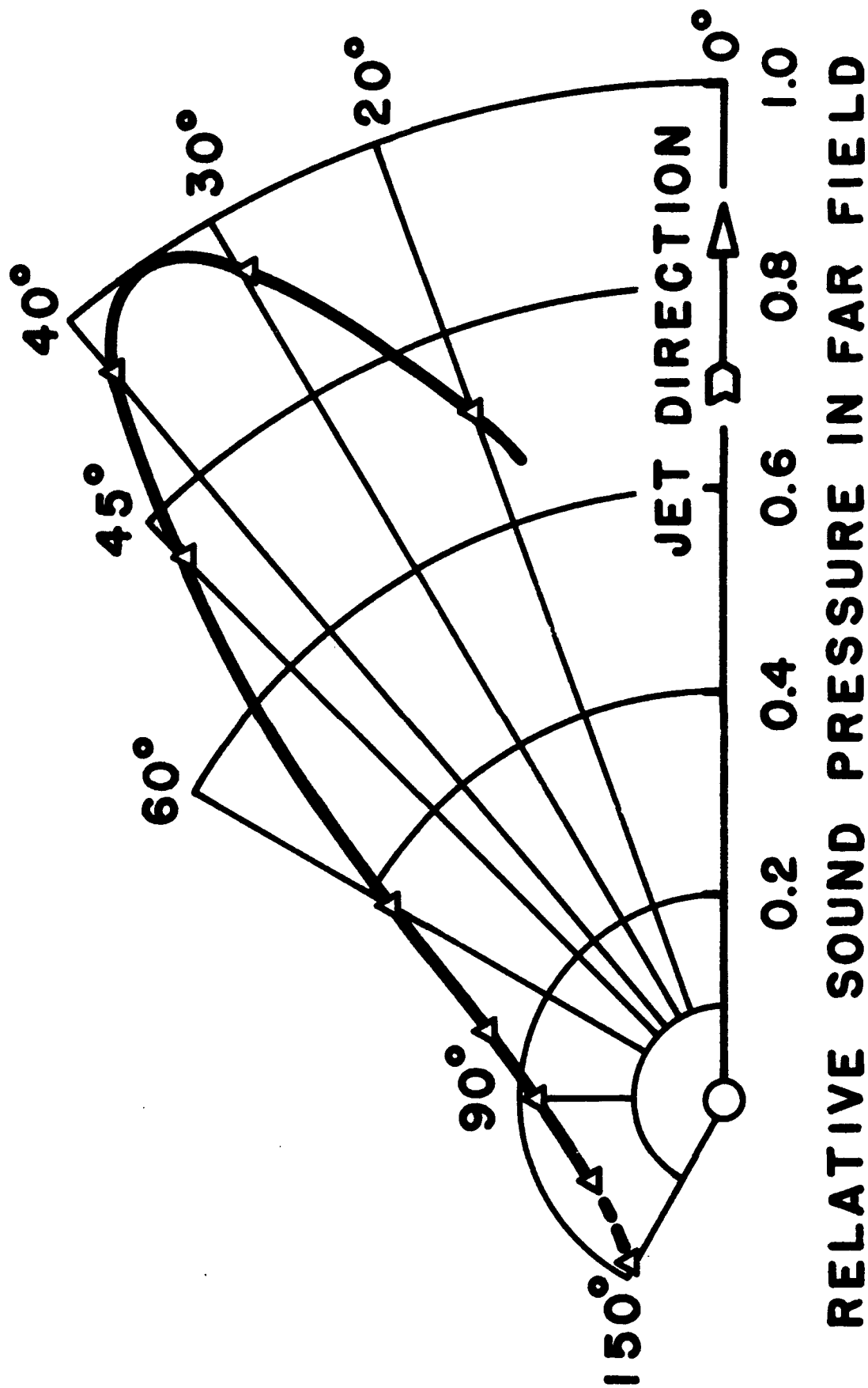


FIGURE 5. DIRECTIONAL DISTRIBUTION OF THE RADIATED SOUND ENERGY FOR THE JET ENGINE HAVING THE POWER SPECTRUM OF FIGURE 4. NOTE THAT THE RADIAL SCALE OF RELATIVE SOUND PRESSURE IS LINEAR.

to source move quicker in the upstream direction than they do in the opposite direction. In fact, for the downstream direction, the time taken is increased by a factor  $1/(1-M_c)$ , where  $M_c$  is the local convection Mach number,  $U_{\text{mean}}/a_a$ , of the local eddy system. For points at an angle  $\theta$  to the jet direction, the factor should be  $1/(1-M_c \cos \theta)$ . Hence so far as the noise intensity of a quadrupole is concerned, a factor  $1/(1-M_c \cos \theta)^4$  arises (effects of two-fold cancellations, squared). Finally, the simple source itself accounts for a similar factor, because of the moving source effect, radiated frequencies being factored by  $1/(1-M_c \cos \theta)$ . All together these factors give  $1/(1-M_c \cos \theta)^6$  to be expected on the radiated noise intensity; and moreover this is in qualitative agreement with the observed directionality of jet noise radiation, which we have reason to believe originates from lateral quadrupoles oriented as shown in Figure 1, together with longitudinal quadrupoles of random orientation. But it is important to note that the increase in the downstream directions always exceeds the reduction in the upstream directions. Hence, the total noise power generated increases, by a large amount at high convection Mach numbers.

Thus, in explaining the characteristic directional pattern of the jet noise, we must accept the consequent fact that the total noise output increases as a function of Mach number, so Equation (4b) should have been instead

$$P \sim \rho_a (\rho_j / \rho_a)^\epsilon (u_f^4 v V / a_a^5) (U_j / a_a)^\alpha G_3 \quad (12b)$$

where  $\alpha$  is not a constant, but increases rather sharply with  $M_c \sim U_j / a_a$ . Over the range of jet exit Mach numbers from 0.5 to 0.9, assuming  $M_c$  to be half of these,  $\alpha$  averages 1.8. At higher exit velocities, the situation is more acute. For convection Mach numbers between 0.5 and 0.9  $\alpha$  averages no less than 12.8. Such convection Mach numbers are not inconceivable for jet engine exhausts, where the Mach number  $U_j / a_a$  often exceeds two.

We had better recognize formally at this stage that the assumptions of similarity we have used are really only first approximations. The effect of all possible departures from this have been wrapped up in the function  $G_3$  of the last equation.

Now we have a problem to resolve. Lighthill's convection idea is the only one that satisfactorily explains the gross bias of the jet radiation in the downstream direction and this necessarily raises the dependence of the overall

noise on jet velocity to a power appreciably greater than eight. To quote Lighthill, "Paradoxically enough, the feature of the experimental results which is the hardest of all to explain . . . is the accuracy with which the acoustic power output varies as  $U^8$ ."

## 7. SOME JET TURBULENCE CHARACTERISTICS

It is now necessary to see how certain measurements of turbulence in jets bear on the foregoing considerations. It will be appreciated that the experimental data, for this particular point of view, is rather fragmentary, and therefore, this section is somewhat tentative.

Along the length of the potential cone, the maximum velocity fluctuations occur at the distance of about a nozzle radius from the axis and the amplitude remains constant.<sup>14, 27</sup> Thus, the assumption  $u \sim x^0$  has some support. The turbulent fluctuations appear to follow  $u \sim U_j^{3/4}$ , very nearly, at important points in the early mixing region<sup>27, 28</sup> (although some assurance concerning instrument calibration seems necessary<sup>29</sup>).

The scale of turbulence,  $l$ , (which is presumably connected to the size of the "noise generating eddies") increases over the length of the potential cone, but much less quickly than proportionally to the distance from the exit. In fact,  $l \sim x^{1/2}$  seems to be the best fit over what data we have.<sup>27, 28</sup>

Measurements of the peak turbulence frequency suggest that  $f \sim U_j$ . Also the peak frequency of certain spectra seems to follow  $f \sim x^{-1}$ . But these are presumably more measures of the "passage frequency" of the eddy system rather than of the true time fluctuations of the convected turbulence.

## 8. DISCUSSION

Use will now be made of the foregoing results to see whether any improvements upon our simple similarity arguments are readily available. It is emphasized that this section is of a highly tentative nature, and should be considered in a speculative vein until data to judge it properly is available. Equation (5) will be modified to read

$$(dP/dx)_A \sim u^4 f^4 v A U_j^\alpha G_4 \quad (13)$$

Here  $U_j^\alpha$  represents convection effects that cannot be justifiably left out. The "constants"  $\rho_a$  and  $a_a$  are incorporated in  $G_4$ , as will be  $D$  which is now abandoned as an independent variable because of inadequate information to

do otherwise. If we make the right assumptions about scaling,  $G_4$  will be constant, and with this understanding it is dropped.

Thus we take  $\alpha = 2$  to represent convection effects at moderate speeds, and the eddy scale  $\ell \sim x^{1/2}$ . We could take  $f \sim U_j/x$  as certain measurements suggest, but this leads to a spectral slope like  $f^{-1/2}$  which has never been observed. Therefore, we shall take the alternative form  $f \sim u/\ell$  which seems to be a more satisfactory measure of the time fluctuations in the turbulence. Then to still have  $P \sim U_j^8$ , we must have  $u \sim U_j^{3/4}$ , which is exactly what has been measured. In this way the high-frequency spectrum becomes

$$(dP/dx)_A \sim U_j^8 x^{1/2} \quad (14a)$$

$$(dP/df)_A \sim U_j^{10 1/4} f^{-4} \quad (14b)$$

with the possibility of

$$f_p \sim U^{<1} \quad (14c)$$

These spectral trends are consistent with several experimental data (whether or not the acoustic power output is weighted toward the end of the potential cone is not easily determined). These results also give the right order of overall acoustic power.

That the peak of the spectrum in all probability emanates from near, or a little downstream of, the end of the potential cone seems to be one of the most firm conclusions one can make. The method holds promise of further development in due course, including applications to the flight case for jet engines (see Reference 30 for a preliminary attempt).

It is interesting to speculate a little upon some of the features not well represented in the simple analyses above: one concerns the steepening of the spectral slopes with speed.

It may be that the highest frequencies emanate from eddies small enough to be effectively immersed in the jet stream, radiating into the jet stream itself without the gross convection effects, so being less dependent on the jet velocity, i. e., the spectral slope should increase with velocity, and possibly frequency, which, of course, is the case. The sound so generated will suffer

a gross diffraction away from the jet axis by the high stream velocity, in addition to any scattering. This seems to be a plausible reason for the highest frequencies displaying maxima at large acute angles to the jet axis,<sup>2</sup> up to at least  $80^\circ$ . It is faintly possible that as the speed increases, the shortening wavelengths of the peak frequencies suffer some reduction of the convection effect, relevant to the observed failure of radiated frequencies to scale proportionally to velocity. It is not yet known how relevant are detailed studies of isotropic turbulence to the jet problem, but some recent work<sup>31</sup> suggests that the highest frequencies of the noise from isotropic turbulence depends on  $u^{10^{1/2}} f^{-3^{1/2}}$ .

With regard to temperature effects, the spectral peak is presumably less affected than the higher frequencies emanating from nearer the jet exit, suggesting some increase in the high-frequency slope for high jet temperatures, with a relatively small influence on the overall noise power. Further, there is reason to believe that the noise power generation is enhanced a little by increased jet temperature.<sup>10</sup>

Consider now the low-frequency emission. The region generating a given frequency moves downstream as the jet speed increases, with both the scale and local velocity increasing. Hence the convection effects will increase, and the higher the velocity, (and so frequency) the faster is that increase. Thus we have a partial explanation for the increase in slope of the low-frequency part of the spectra with the speed, {associated with  $\alpha$  of Equation (13) being velocity dependent}. The low-frequency slope, however, is generally less steep than our simple arguments suggest. (One reason may be that the spectra emanating from slices of the jet individually are less steep than  $f^2$ .)

## 9. ROCKET NOISE

After it was realized that the efficiency with which jet noise is generated depends upon the fifth power of Mach number  $U_j/a$ , it became a matter of considerable interest to learn what happens for rockets. Clearly that fifth power relationship could not continue indefinitely, since otherwise at a Mach number of six or so all the kinetic energy of the jet stream would be converted to acoustic energy. Such a situation could not exist, of course; it turns out that so far the acoustic power appears to be limited to about one-hundredth of the rocket power, which nevertheless, is an extremely powerful source. The departure from the low-speed characteristics is shown very clearly in

Figure 6, using data from Reference 32. The spectrum is like the averaged curve of Figure 4, except that the low frequencies may be slightly more predominant.<sup>24</sup>

At such high speeds the simple theoretical arguments presented in this paper break down, since the convection Mach number will be supersonic for much of the jet, and in any case, it is necessary to consider the effects of Mach number on the structure of the turbulence and on the jet spread. We shall not speculate on these at present, but just comment that much of the noise generated leaves the jet at angles so as to suggest that it is associated with the "shockwaves of supersonic eddies".

#### 10. CONCLUDING REMARKS

It has been shown how simple arguments can yield much useful information about jet noise, once its fundamental nature is realized. Although the most simple ideas yield results in remarkable agreement with experiment, it is necessary to embellish the method to account for certain other features. The most important of these concern: (a) the characteristic directionality of jet noise; (b) the failure of the peak frequency to scale with velocity; (c) the insensitivity of the overall power to jet temperature (d) the failure of present methods at very high speeds. The discussion illustrated how present information can be used to speculate upon some of these points; clearly much still remains to be done before the picture is complete. Attention is focussed upon the need for detailed information about the transitional region of the jet flow because it, in all probability, accounts for the spectral peak--and it is not amenable to similarity arguments!

#### 11. ACKNOWLEDGMENTS

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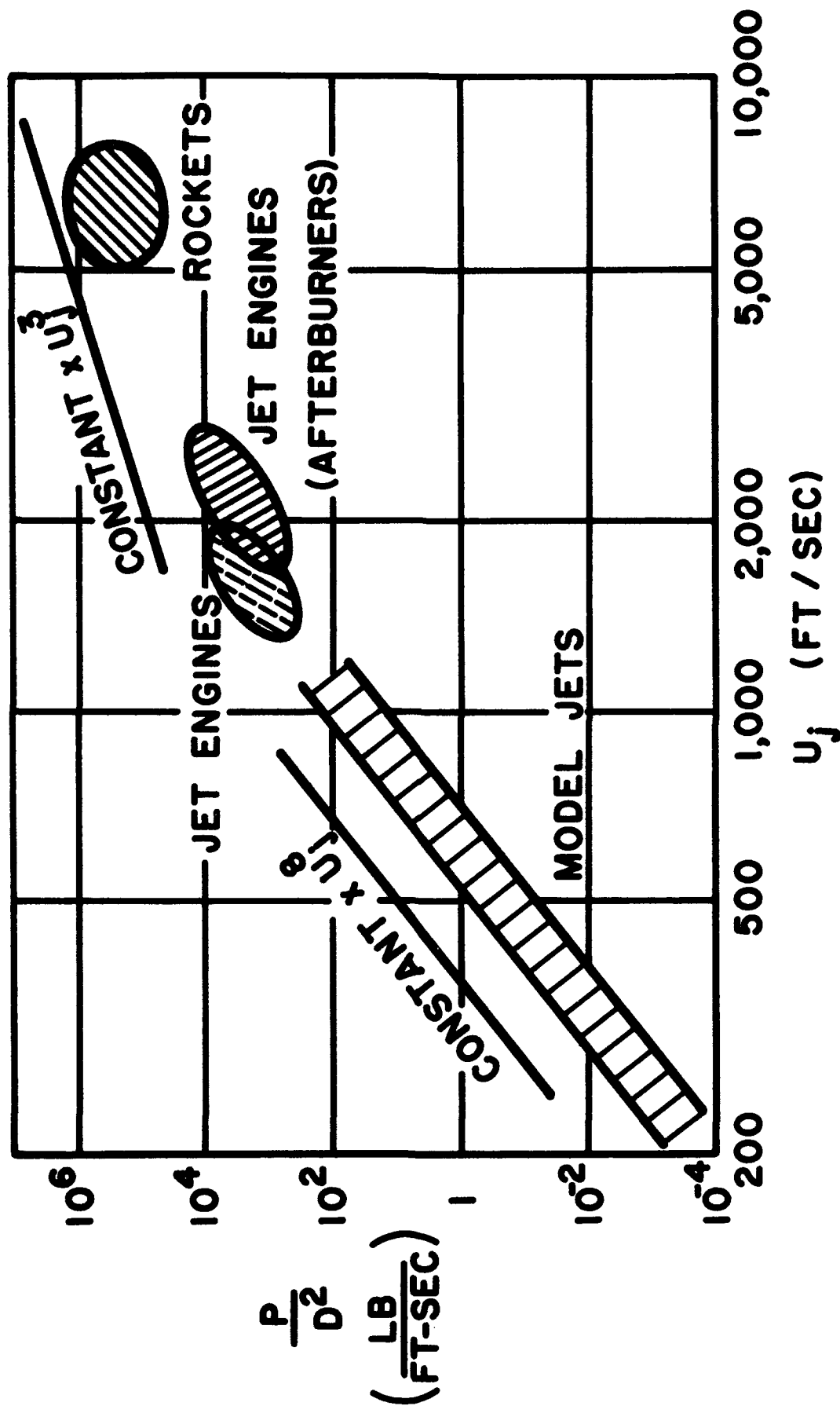


FIGURE 6. HIGH-SPEED JETS HAVE A NOISE POWER FAR BELOW THAT OBTAINED BY PRODUCING LOW-SPEED NOISE RELATIONSHIPS TO THOSE SPEEDS. (THE EFFECT IF DIAMETER HAS BEEN FACTORED OUT.)

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